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Capacitance in a radio-frequency-SQUID magnetometer

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Abstract. We use a discrete-field model for the quantum evolution of distributed superconducting circuits to calculate the effective capacitance for a radio-frequency-SQUID magnetometer: a large-capacitance superconducting ring containing an ultra-small-capacitance weak link. We demonstrate that the self-capacitance of the weak link can dominate the behaviour of the circuit.

The radio-frequency- (rf-) SQUID magnetometer is the simplest superconducting circuit containing a non-linear circuit element (a thick superconducting ring containing a Josephson weak link [1, 2]) and is an ideal system for studying the appearance of quantum mechanical behaviour in macroscopic circuits (for recent reviews see [3, 4, 5, 6]). A particular series of experiments has provided excellent evidence for the existence of discrete energy levels in the rf-SQUID ring (corresponding to the stationary solutions of the Schrödinger equation with a macroscopic Hamiltonian) [7]. In such a circuit, the capacitance plays the role of an effective mass for the total magnetic flux enclosed within the ring. The appearance of a discrete set of energy levels is therefore associated with a small capacitance (i.e. a small ‘mass’). The best agreement between the theory and experimental results comes from an effective capacitance of around $C \simeq 10^{-16}$ F [7]. This value is significantly lower than would be expected from geometrical arguments [8], and has led to the proposal of other models for the experimental behaviour [9]. In this paper we demonstrate that the effective capacitance of an rf-SQUID ring can be determined by the small self-capacitance of the weak link, even in the presence of a large mutual capacitance associated with the thick ring structure.

It is well known that (low temperature) superconductivity is associated with a broken gauge symmetry of the electromagnetic field [10, 11] and this can be used to construct a distributed circuit Hamiltonian for the transverse and longitudinal fields [12]. The main advantage of this approach is that it can be used to describe complex (spatially distributed) circuits in a consistent manner, avoiding problems associated with the confusion of longitudinal fields (charge–phase) and transverse fields (electric flux–magnetic flux) [5]. However, it has also been shown that this approach also leads to an additional (non-geometric) capacitance, associated with the superconducting condensate itself [12], and that this capacitive energy can be significant for a Josephson weak-link device [1, 2].

We start with a Lagrangian for the symmetry broken electromagnetic field. We choose the simplest case which gives the required macroscopic behaviour [10, 12],

$$L = \int \mathrm{d}V \left[\frac{\epsilon_0 \boldsymbol{E}^2 - \mu_0^{-1} \boldsymbol{B}^2}{2} + \frac{\mathcal{N} \nu q^2}{\hbar^2} \left(\dot{\phi} + A^0 \right)^2 - \frac{\mathcal{N} \gamma q^2}{\hbar^2} \left(\nabla \phi - \boldsymbol{A} \right)^2 + a\mathcal{N} - \frac{b\mathcal{N}^2}{2} \right] \quad (1)$$

where $q = 2e$; the electric field $\boldsymbol{E} = -\nabla A^0 - \frac{\partial \boldsymbol{A}}{\partial t}$ and magnetic field $\boldsymbol{B} = (\nabla \times \boldsymbol{A})$ are given by the scalar and vector potentials, A^0 and $\boldsymbol{A}$; and $\mathcal{N}$ is non-zero inside the superconductor, but vanishes outside. (We take the permittivity and permeability of the material to be the same as those of free space, ϵ_0 and μ_0 respectively.) This Lagrangian can also be obtained from the time-dependent Ginzburg–Landau model [13, 14], where the order parameter has been expressed as $\Phi = \sqrt{\mathcal{N}} \exp(\mathrm{i}q\phi/\hbar)$ and where the number density, $\mathcal{N}$, is taken to be a smooth, static classical field (for simplicity). a and b are then found to be the usual Ginzburg–Landau coefficients, with $\gamma = \hbar^2/2m_e$ and $\nu = 3\gamma/v_F^2$ (where v_F is the Fermi velocity of the electrons in the (non-superconducting) material) [13, 14].

Unfortunately, not all of the fields have a conjugate momemtum. This makes the construction of a Hamiltonian, and the subsequent imposition of canonical commutation relations, more complicated than it might otherwise be [15]. The classical equations for A^0 and $\mathcal{N}$ form constraints, which (together with the usual gauge constraints) must be imposed before the fields can be quantized. Taking the variational derivative of the Lagrangian density with respect to A^0 and $\mathcal{N}$, we obtain

$$\frac{\delta \mathcal{L}}{\delta A^0} = \epsilon_0 \nabla \cdot \left(\nabla A^0 + \frac{\partial \boldsymbol{A}}{\partial t} \right) - \frac{2\mathcal{N} \nu q^2}{\hbar^2} \left(\frac{\partial \phi}{\partial t} + A^0 \right) = -\epsilon_0 \nabla \cdot \boldsymbol{E} + \rho = 0 \tag{2}$$

where the charge density is given by

$$\rho = -\frac{\delta \mathcal{L}_m}{\delta A^0} = -\frac{2\mathcal{N} \nu q^2}{\hbar^2} \left(\frac{\partial \phi}{\partial t} + A^0 \right) \tag{3}$$

and $\mathcal{L}_m$ is the material part of the Lagrangian density (i.e. the symmetry breaking terms only); and

$$\frac{\delta \mathcal{L}}{\delta \mathcal{N}} = \frac{\hbar^2}{4\nu q^2} \left(\frac{\rho}{\mathcal{N}} \right)^2 - \frac{\gamma q^2}{\hbar^2} (\nabla \phi - \boldsymbol{A})^2 + a - b\mathcal{N} = 0 \tag{4}$$

which becomes $\mathcal{N} \simeq a/b$, if a and b are sufficiently large [13]. The first constraint is simply Gauss' law. The second constraint arises because we have taken the number density to be a smooth, static classical field (neglecting the time and spatial derivatives). Whilst this is not strictly necessary (the quantum dynamics of this degree of freedom can also be included), the fact that the number density is often associated with topological excitations (vortex lines) means that it would be very complicated to describe with a quantum mechanical Hamiltonian. Provided we remain outside the regimes where we would expect these topological excitations to occur (near the critical temperatures and at relatively high magnetic fields), this approximation should be good.

Imposing these constraints, together with the gauge constraints for the Coulomb gauge [15],

$$\nabla \cdot \boldsymbol{A} = 0 \tag{5}$$

and

$$\epsilon_0 \nabla^2 A^0 = \nabla \cdot \boldsymbol{p} \tag{6}$$

(where $\boldsymbol{p} = -\epsilon_0 \boldsymbol{E}$ is the momentum conjugate to $\boldsymbol{A}$), we obtain a Hamiltonian of the form

$$H = \int \mathrm{d}V \left[\frac{\epsilon_0 \boldsymbol{E}^{\perp 2}}{2} + \frac{(\nabla \times \boldsymbol{A})^2}{2\mu_0} - \frac{\rho\, \Delta^{-1} \rho}{2\epsilon_0} + \frac{1}{2} \left(\frac{\hbar^2}{2\nu q^2} \right) \frac{\rho^2}{\mathcal{N}} + \frac{\mathcal{N} \gamma q^2}{\hbar^2} (\nabla \phi - \boldsymbol{A})^2 \right] \tag{7}$$

where $\boldsymbol{E}^{\perp}$ is the transverse part of the electric field and $\Delta \equiv \nabla^2$. The Hamiltonian contains three fields: the two transverse electromagnetic fields (the divergenceless parts of $\boldsymbol{A}$) and the symmetry breaking field (ϕ, identified with the longitudinal electromagnetic field); and their conjugate momenta, $p_{\boldsymbol{A}} = -\epsilon_0 \boldsymbol{E}^\perp$ and $p_\phi = -\rho$ respectively. The first three terms are the usual electromagnetic field energies (transverse fields and Coulomb interaction). The last two terms are responsible for the broken symmetry, giving rise to a self-capacitance for the superconducting condensate and the exclusion of magnetic flux in the bulk material respectively.

We then introduce the canonical commutation relations for the fields and their conjugate momenta, to obtain

$$[E_i^\perp(x), A_j(y)] = \frac{\mathrm{i}\hbar}{\epsilon_0}\Pi_{ij}\delta(x-y) \tag{8}$$

where $\Pi_{ij} = \delta_{ij} - \Delta^{-1}\partial_i\partial_j$; and

$$[\rho(x), \partial_i\phi(y)] = \mathrm{i}\hbar\partial_i\delta(x-y). \tag{9}$$

We write the second commutation relation between the charge density and the gradient of the field ϕ, because ϕ is an angular operator (period $\Phi_0 = h/q$) and can only appear in a periodic function or as a derivative [16]. This is due to the requirement that the physical charge contained within any closed surface be an integer multiple of $q = 2e$, or superposition thereof (from Gauss' law and the quantization of real charge in the condensate). We note that, due to the broken symmetry, the longitudinal electromagnetic field is now a real physical field [17] (with corresponding physical excitations [10]). In a normal conductor, the symmetry is unbroken and the longitudinal field has no physical excitations.

Given a particular geometry, using this field Hamiltonian and the corresponding commutation relations, it should be possible to solve for all of the circuit modes of a simple superconducting ring. In this paper, we restrict ourselves to the discussion of one simple example: an rf-SQUID ring [2]. In this case, we can simplify consideration by assuming that the circuit consists of a uniform circular ring (whose cross-sectional dimensions are 'thick' compared to the magnetic penetration depth [13], but 'thin' compared to the radius of the ring [12]) which contains a point contact weak link [2]. If we further restrict ourselves to the two lowest-energy modes of the ring, we can describe the system in terms of three segments: two for the bulk ring and one for the weak link (see figure 1). We note that, in contrast with the model given in [9], the Hamiltonian requires only one weak-link segment.

In this case, the appropriate Hamiltonian is given by [12]

$$H = \left(\frac{Q_2^2}{2C_2^{(t)}} + \frac{\Phi_2^2}{2\Lambda_2}\right) + \left(\frac{Q_3^2}{2C_3^{(t)}} + \frac{\Phi_n^3}{2\Lambda_3}\right) + \frac{q_2^2}{2C_{22}} + \frac{q_2 q_3}{2C_{23}} + \frac{q_3^2}{2C_{33}}$$
$$+ \frac{qI_{c_3}}{2\hbar}(\Phi_3 - \Delta\phi_3)^2 + \frac{qI_{c_2}}{2\hbar}(\Phi_2 + \Delta\phi_1 + \Delta\phi_3)^2 - \frac{\hbar I_{c_1}}{q}\cos\left(\frac{2\pi\,\Delta\phi_1}{\Phi_0}\right)$$

where the local discrete-field operators are defined by

$$q_n = \int_{V_n} \mathrm{d}V\rho(x) \tag{10}$$

$$\Delta\phi_n = \int_{C_n} \mathrm{d}\boldsymbol{l}\cdot\nabla\phi(x) = \phi(x_{n+1}) - \phi(x_n) \tag{11}$$

and

$$Q_n = \epsilon_0 \int_{S_{Qn}} \mathrm{d}\boldsymbol{S}\cdot\boldsymbol{E}^\perp(x) \tag{12}$$

$$\Phi_n = \int_{S_{\Phi n}} \mathrm{d}\boldsymbol{S}\cdot\boldsymbol{B}(x) = \oint_{\partial S_{\Phi n}} \mathrm{d}\boldsymbol{l}\cdot\boldsymbol{A}(x). \tag{13}$$

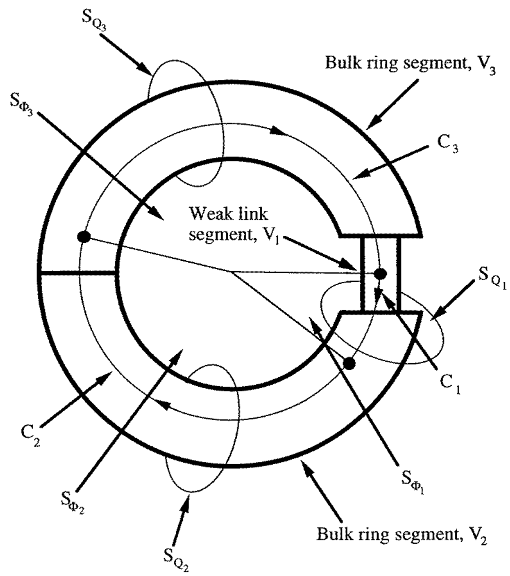

Figure 1. Schematic diagram of the simple radio-frequency SQUID ring; divided into three segments, showing definitions of discrete field variables.

(The volumes (V_n), curves (C_n) and open surfaces (S_{Qn} and $S_{\Phi n}$) are shown in figure 1.)

We have used the fact that the total charge and total phase change around the ring are constants to remove $q_1 = -q_2 - q_3$ and $\Delta\phi_2 = m\Phi_0 - \Delta\phi_1 - \Delta\phi_3$ (m is an integer). We have also assumed that the transverse fields around the weak link (given by Φ_1 and Q_1) are only very weakly coupled to the excitations of the bulk ring, so that they can also be removed. The remaining non-zero commutation relations are then

$$[Q_2, \Phi_2] = [Q_3, \Phi_3] = \mathrm{i}\hbar \tag{14}$$

and

$$[q_3, \Delta\phi_3] = -[q_2, \Delta\phi_1] = \mathrm{i}\hbar. \tag{15}$$

To obtain this Hamiltonian, we parametrize the field energies in terms of effective circuit elements. The transverse electromagnetic fields for each segment are given by an effective (transverse) capacitance ($C_n^{(t)}$) and a geometrical inductance (Λ_n). The longitudinal field energies are given in terms of two self-capacitances for the two halves of the bulk ring (C_{22} and C_{33}) and one mutual capacitance (C_{23}), but each of these will contain terms due to the usual geometrical capacitances (from the Coulomb energy) and from the self-capacitance of the condensate:

$$\frac{1}{C_{22}} = (C_{(l)}^{-1})_{11} - (C_{(l)}^{-1})_{12} - (C_{(l)}^{-1})_{21} + (C_{(l)}^{-1})_{22} + \frac{1}{C_1^{(sc)}} + \frac{1}{C_2^{(sc)}}$$

$$\frac{1}{C_{23}} = 2(C_{(l)}^{-1})_{11} - (C_{(l)}^{-1})_{12} - (C_{(l)}^{-1})_{21} - (C_{(l)}^{-1})_{13} - (C_{(l)}^{-1})_{31} + (C_{(l)}^{-1})_{23} + (C_{(l)}^{-1})_{32} + \frac{2}{C_1^{(sc)}}$$

$$\frac{1}{C_{33}} = (C_{(l)}^{-1})_{11} - (C_{(l)}^{-1})_{13} - (C_{(l)}^{-1})_{31} + (C_{(l)}^{-1})_{33} + \frac{1}{C_1^{(sc)}} + \frac{1}{C_3^{(sc)}}$$

where $C_{(l)}$ is the effective capacitance matrix due to the Coulomb interaction, and $C_n^{(sc)}$ is the self-capacitance of the condensate for the nth element. To obtain an expression for this self-capacitance we look at the terms which couple the transverse and longitudinal fields. The $(\nabla\phi - \boldsymbol{A})^2$ term in the field Hamiltonian can be rewritten as,

$$(\nabla\phi - \boldsymbol{A})^2 = \frac{\hbar^2}{q^2}\left\{\left(\nabla + \frac{\mathrm{i}q\boldsymbol{A}}{\hbar}\right)\mathrm{e}^{(-\mathrm{i}q\phi/\hbar)}\right\}\cdot\left\{\left(\nabla - \frac{\mathrm{i}q\boldsymbol{A}}{\hbar}\right)\mathrm{e}^{(\mathrm{i}q\phi/\hbar)}\right\} \tag{16}$$

which becomes (for small Δx)

$$\frac{\mathcal{N}\gamma q^2(\Delta V_n)}{\hbar^2}(\nabla\phi - \boldsymbol{A})^2 \simeq \frac{\gamma\mathcal{N}_n\sigma_n}{(\Delta x)}\left[2 - 2\cos\left(\frac{2\pi\,\Delta\phi_n}{\Phi_0}\right) - \left(\frac{2q\Phi_n}{\hbar}\right)\sin\left(\frac{2\pi\,\Delta\phi_n}{\Phi_0}\right) + \frac{q^2\Phi_n^2}{\hbar^2}\right] \tag{17}$$

where σ_n is the cross-sectional area of the nth ring segment, i.e. $(\Delta V_n) = \sigma_n(\Delta x)$ and $\mathcal{N}_n$ is the number density on the nth segment. This form not only produces the familiar Josephson tunnelling terms [1, 2], it also preserves the angular nature of the longitudinal field. For large Δx (the bulk ring segments), we assume that the angular nature of ϕ will not be important and make a quadratic approximation $(\nabla\phi - \boldsymbol{A})^2 \propto (\Delta\phi_n - \Phi_n)^2$.

Comparing (17) to the known expression for the energy of a Josephson device (the amplitude of the cosine term is equal to $\hbar I_c/q$) we obtain

$$I_{c_n} = \frac{q\hbar\mathcal{N}_n\sigma_n}{m_e(\Delta x)} \tag{18}$$

where I_{c_n} is the critical current of the nth segment and (Δx_n) is its length. Combining this with the expression for ν given above, the condensate self-capacitance is given by [12],

$$C_n^{(sc)} = \frac{3qI_{c_n}(\Delta x_n)^2}{\hbar v_F^2}. \tag{19}$$

Next, we let the critical currents for the bulk ring (I_{c_2} and I_{c_3}) become very large, so that $\Phi_2 \simeq -(\Delta\phi_1 + \Delta\phi_3)$ and $\Phi_3 \simeq \Delta\phi_3$. Using these as constraints, we rewrite the Hamiltonian (10) as a (classical) Lagrangian, introduce the constraints and find the new conjugate momenta for the operators Φ_2 and Φ_3. We then requantize the system by imposing new canonical commutation relations.

Doing this procedure, we obtain the new conjugate momenta,

$$p_{\Phi_2} = -Q_2 - q_2 = -\tilde{Q}_2 \tag{20}$$

$$p_{\Phi_3} = -Q_3 - q_2 - q_3 = -\tilde{Q}_3 \tag{21}$$

and making the transformation

$$\Phi = (\Phi_2 + \Phi_3) \qquad Q = \tfrac{(\tilde{Q}_2 + \tilde{Q}_3)}{2}$$

$$\Delta\Phi = (\Phi_2 - \Phi_3) \qquad \Delta Q = \tfrac{(\tilde{Q}_2 - \tilde{Q}_3)}{2}$$

we obtain the new Hamiltonian,

$$H = \left[\frac{Q^2}{2C} + \frac{\Phi^2}{2\Lambda} - \frac{\hbar I_{c_1}}{q}\cos\left(\frac{2\pi\Phi}{\Phi_0}\right)\right] + \frac{Q(\Delta Q)}{2C'} + \left[\frac{(\Delta Q)^2}{2C''} + \frac{(\Delta\Phi)^2}{2\Lambda}\right] \tag{22}$$

and commutation relations

$$[Q, \Phi] = [\Delta Q, \Delta\Phi] = \mathrm{i}\hbar. \tag{23}$$

The first part of (22) is the usual Hamiltonian for a lumped component rf-SQUID ring [2, 3], involving the electric flux (Q) and the total enclosed magnetic flux (Φ). The last part is the next-highest-energy electrodynamic mode of the ring, coupled to the lowest energy mode, via an effective capacitance C'. (Since the ring is uniform, we take $\Lambda_2 = \Lambda_3 = \Lambda/2$.)

From (22), we see that the capacitive energy scales are set by the three capacitances: C, C' and C''.

$$C = \frac{(ad-bc)^2}{\left(\frac{d^2}{C'_{22}} - \frac{cd}{C'_{23}} + \frac{c^2}{C'_{33}}\right)} \tag{24}$$

$$C' = \frac{(ad-bc)^2}{\left(-\frac{2bd}{C'_{22}} + \frac{(ad+bc)}{C'_{23}} - \frac{2ac}{C'_{33}}\right)} \tag{25}$$

$$C'' = \frac{(ad-bc)^2}{\left(\frac{b^2}{C'_{22}} - \frac{ab}{C'_{23}} + \frac{a^2}{C'_{33}}\right)} \tag{26}$$

where

$$a = 1 + \frac{C_3^{(t)}}{4C_{23}} + \frac{C_2^{(t)}}{2}\left(\frac{1}{C_{22}} - \frac{1}{2C_{23}}\right) \qquad b = \frac{1}{2} + \frac{C_3^{(t)}}{2C_{33}} + \frac{C_2^{(t)}}{2}\left(\frac{1}{2C_{23}} - \frac{1}{C_{33}}\right)$$

$$c = -\frac{C_3^{(t)}}{4C_{23}} + \frac{C_2^{(t)}}{2}\left(\frac{1}{C_{22}} - \frac{1}{2C_{23}}\right) \qquad d = -\frac{1}{2} - \frac{C_3^{(t)}}{2C_{33}} + \frac{C_2^{(t)}}{2}\left(\frac{1}{2C_{23}} - \frac{1}{C_{33}}\right)$$

$$C'_{22} = \left[\frac{1}{C_{22}} + \frac{C_3^{(t)}}{4C_{23}^2} + C_2^{(t)}\left(\frac{1}{C_{22}} - \frac{1}{2C_{23}}\right)^2\right]^{-1}$$

$$C'_{23} = \left[\frac{1}{C_{23}} + \frac{C_3^{(t)}}{C_{33}C_{23}} + 2C_2^{(t)}\left(\frac{1}{C_{22}} - \frac{1}{2C_{23}}\right)\left(\frac{1}{2C_{23}} - \frac{1}{C_{33}}\right)\right]^{-1}$$

$$C'_{33} = \left[\frac{1}{C_{33}} + \frac{C_3^{(t)}}{C_{33}^2} + C_2^{(t)}\left(\frac{1}{C_{33}} - \frac{1}{2C_{23}}\right)^2\right]^{-1}.$$

For the rf-SQUID structure used in the experiments under discussion [7], a macroscopic Niobium ring (radius $\sim$ 5 mm) containing a point-contact weak link [2], the geometrical capacitances associated with the block structure are generally $C_{(l)} \sim 10^{-12}$ F [8], compared with a self-capacitance $C^{(sc)} \sim 5 \times 10^{-17}$–$5 \times 10^{-15}$ F for the weak link [12] (using $\Delta x \sim 0.1$–1 μm, $I_c \sim 1$ μA and $v_F \sim 10^6$ m s^{-1}). Taking these values, and using an estimate for the transverse field energy (based on a uniform, circular ring, $C^{(t)} \sim 10^{-15}$ F [18]), we can calculate the effective capacitances (C, C' and C'') as functions of the self-capacitance of the weak link ($C^{(sc)}$) (see figure 2). For very small-capacitance weak links, the effective capacitance is dominated by the transverse fields and the weak-link capacitance. This is in contrast to the situation found in normal conductors, where the large mutual capacitance of the block would be expected to dominate the behaviour of the system [8], and indeed, this is the case as the symmetry breaking term disappears, $C^{(sc)} \to \infty$.

In general, we find that the small self-capacitance of the weak link couples in series with the capacitance due to the Coulomb term, and that this combination couples in parallel with the effective capacitance of the transverse electric field. The precise value for the capacitances will depend on the geometry of the block and the critical current of the weak link, but an effective capacitance of the order of 10^{-15}–10^{-16} F is not unreasonable.

In conclusion, we have taken a field theoretical model for the quantum evolution of the superconducting condensate [12] and applied it to a particular system which is of great

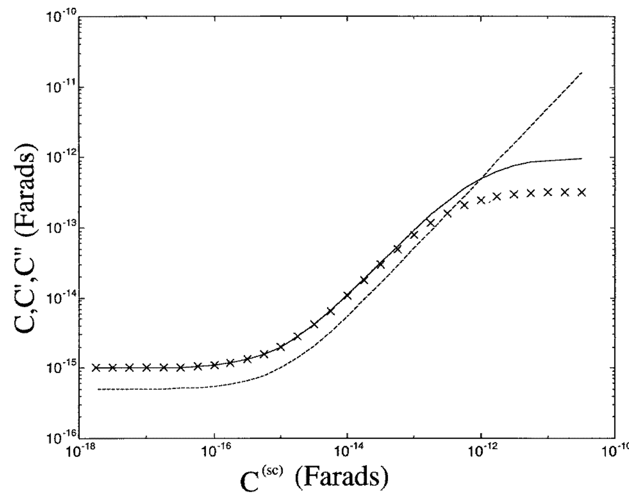

Figure 2. Effective capacitances C (solid line), C' (dashed line) and C'' (crosses) for a macroscopic rf-SQUID ring (parameters given in the text).

experimental interest: the rf-SQUID magnetometer. We have shown that the small self-capacitance of a weak link device can dominate the behaviour of a macroscopic ring, even in the presence of a large geometric capacitance. This result is in contrast to the behaviour predicted in some other models for the rf-SQUID magnetometer which neglect the intrinsic self-capacitance of the condensate [8, 9]. It is, however, in agreement with experimental results, using very small-capacitance, point-contact weak-link devices [7].

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